



# Modelling plasticity

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# FLAC3D

## Constitutive Models for *FLAC* and *FLAC3D*

### *Elasticity models:*

Isotropic  
Transversely isotropic  
Orthotropic

### Built-in Models

### *Plasticity models:*

Drucker-Prager  
Mohr-Coulomb  
Ubiquitous-joint  
Caniso  
Strain-hardening/softening  
Bilinear strain-hardening/softening/ubiquitous-joint  
Double-yield  
Modified Cam-clay  
Hoek-Brown  
Cysoil – friction hardening, with elliptical cap  
Chsoil – simplified Cysoil (alternative to Duncan-Chang)  
PH - plastic hardening  
Swell

### *Dynamic Liquefaction models:*

Finn (Martin et al., 1975) model  
Bryne, 1991 model

### *Creep models:*

Viscoelastic  
Burger's substance viscoelastic  
Two-component power law  
Reference creep formulation (WIPP)  
Burger-creep/Mohr-Coulomb viscoplastic  
Two-component power law/Mohr-Coulomb viscoplastic  
Two-component power law/Mohr-Coulomb viscoplastic with ubiquitous joints  
WIPP-creep/Drucker-Prager viscoplastic  
Crushed-salt

### User-defined Models\*

### *Elasticity models:*

Hyperbolic elastic  
Duncan-Chang, 1980

### *Plasticity models:*

NorSand  
Jardine et al., 1986  
Manzari-Dafalias, 1997  
Kleine et al., 2006  
Concrete hydration  
vonWolffersdorff hypo-plastic  
UBCHyst  
JointedRock

### *Dynamic Liquefaction models:*

UBCSAND  
UBCTOT  
PM4Sand  
Wang, 1990  
Roth et al., 2001  
NTUA-SAND  
SANISAND

### *Creep models:*

Minkley viscoplastic  
Hein-crushed salt  
Salzer creep  
Lubby2 creep

\*partial list of models created by  
(or developed for) code users

# Elastic VS plastic

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## Elasticity

- Describes the behavior of material in which after discontinuing the applied stress the entire deformation remains reversible; the relationship between stress and strain is usually linear.

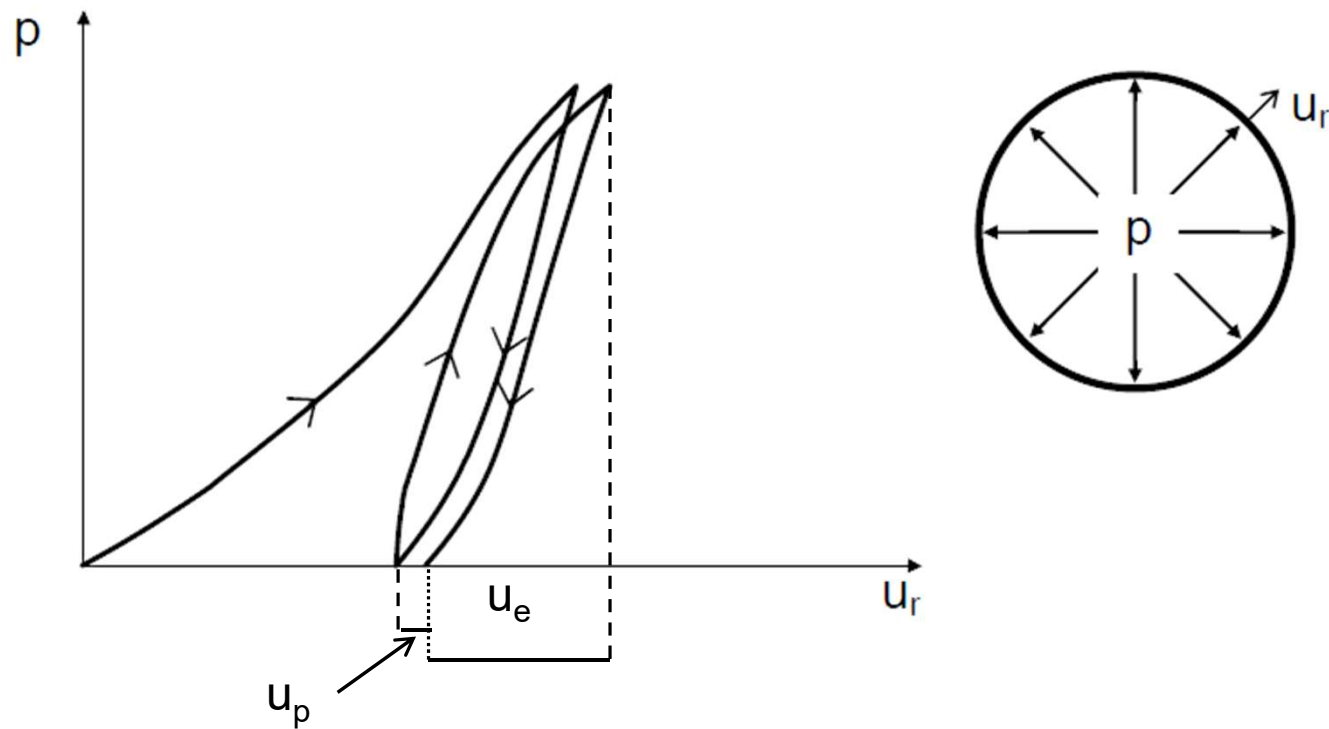
## Plasticity

- Describes a behavior of material which can deform continuously without collapsing under stresses surpassing the one necessary to cause yielding.

# Deformation

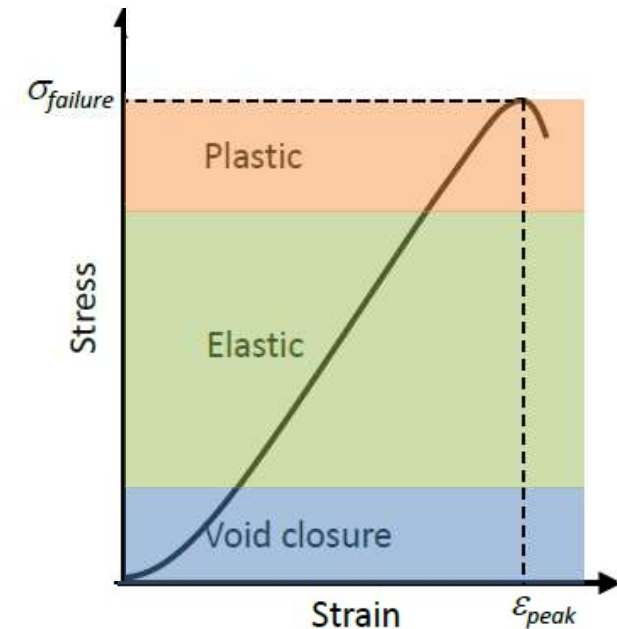
Elastic and plastic deformations

- Reversible and permanent



# Deformation behaviour

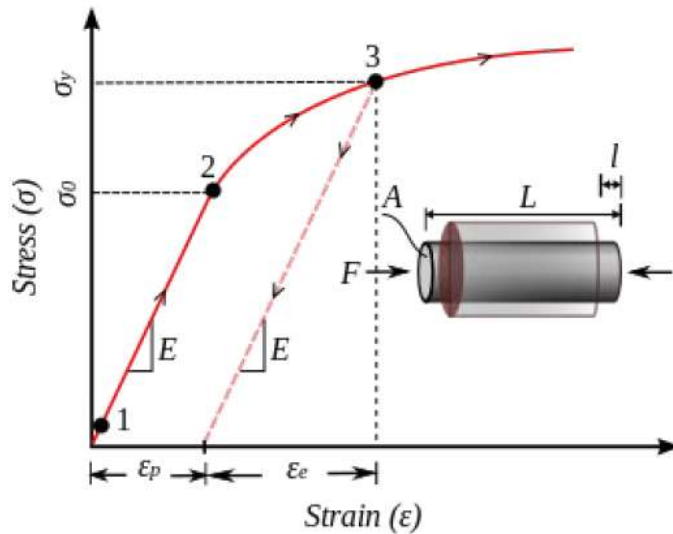
- The stress-strain curve has 3 components before failure:
  1. Void closure – where the voids in the specimen close
  2. Elastic component – elastic deformation
  3. Plastic deformation – permanent deformation
  
- Each of the above 3 component play an important role in failure process of rock specimen



# Calculating plasticity

$$\Delta\varepsilon = \Delta\varepsilon^p + \Delta\varepsilon^e$$

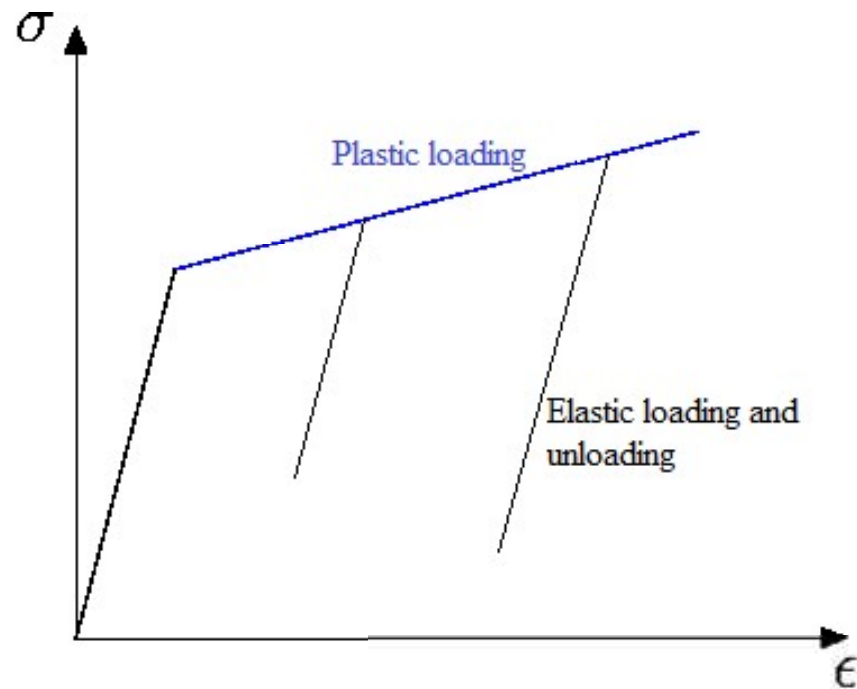
where  $\Delta\varepsilon^e$  is the incremental elastic strain and  $\Delta\varepsilon^p$  is the incremental plastic strain



# Calculating plasticity

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- Active only during plastic loading



# Flow rule

– what happens after we reach the yield criteria?

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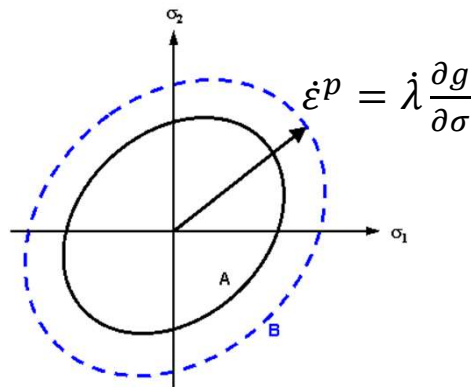
- A constitutive relation for the plastic strain increment
  - Associated

$$g(\sigma) = f(\sigma)$$

- Non associated

$$g(\sigma) \neq f(\sigma)$$

Where  $g$  is the plastic potential function

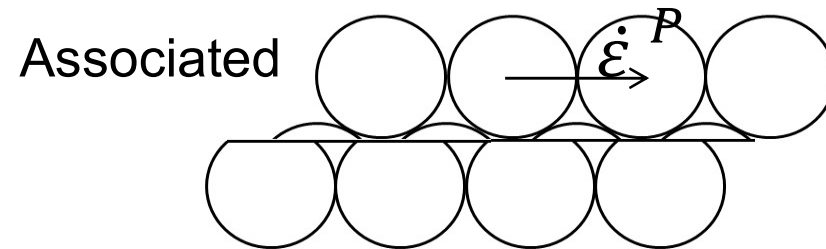
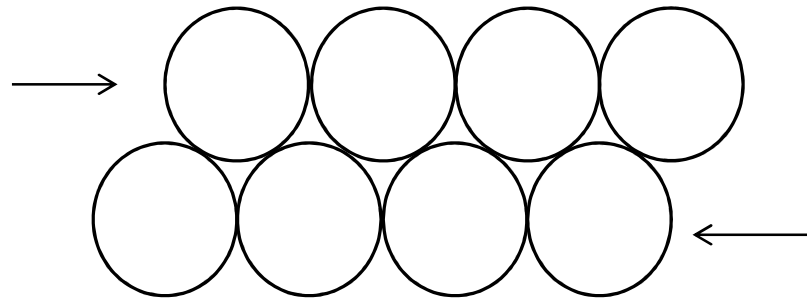




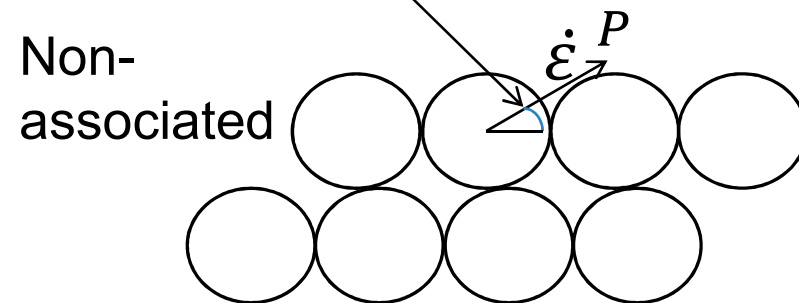
# Flow rule

– what happens after we reach the yield criteria?

If the material has a friction angle = 0



Dilatation angle



# FLAC3D – Mohr-Coulomb model

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Initial yield surface is given by

$$f^i = -\sigma_1 + \sigma_3 N_\phi - 2c\sqrt{N_\phi} \text{ where}$$

$$N_\phi = \frac{1+\sin \phi}{1-\sin \phi} \text{ and } \phi \text{ is the internal friction angle}$$

The plastic potential is given by

$$g^s = -\sigma_1 + \sigma_3 N_\psi \text{ where}$$

$$N_\psi = \frac{1+\sin \psi}{1-\sin \psi} \text{ and } \psi \text{ is the dilatation angle}$$

# FLAC3D – Mohr-Coulomb model

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The Mohr-Coulomb model thus always relates to non-associated flow and the post-peak behavior is controlled by the dilatation angle (as well as any softening criteria applied)