ITASCA™

Theory and background 2

Implicit & Explicit solution

- Implicit formulation (FEM & FDM)
  - Solving of system of equations in every time instance
  - Replacement of *df/dt* with Δt in equilibrium equation
  - Requires iteration, large arrays, memory-intensive
- Explicit formulation (FEM & FDM)
  - Time-stepping – new values in one node calculated from nearby nodes
  - Replacement of *df/dt* with Δt in the equation of motion (F=ma)
  - Non-linear material, large deformations, lower memory requirements

Methods Compared

Explicit, time-marching

1. Can follow nonlinear laws without internal iteration, since displacements are “frozen” within constitutive calculation.
2. Solution time increases as $N^{3/2}$ for similar problems.
3. Physical instability does not cause numerical instability.
4. Large problems can be modeled with small memory, since matrix is not stored.
5. Large strains, displacements and rotations are modeled without extra computer time.

Implicit, static

1. Iteration of the entire process is necessary to follow nonlinear laws
2. Solution time increases with N^2 or even N^3.
3. Physical instability is difficult to model.
4. Large memory requirements, or disk usage.
5. Significantly more time needed for large strain models.

Strengths & Limitations

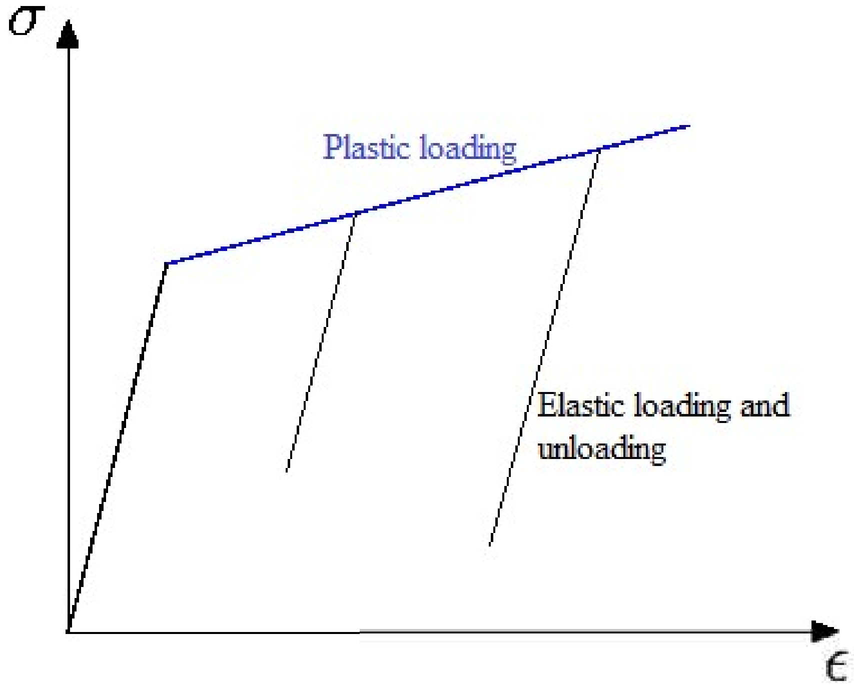
The explicit solution scheme used in FLAC3D enables the following problems to be solved most efficiently:

1. Strongly nonlinear systems, with extensive yield and large strain.
2. Systems in which localization occurs.
3. Systems that embody complex interactions, or which need special user-defined conditions or material models.

Disadvantages are:

1. Slow execution (compared to – say – finite elements) for linear (or well-behaved) systems.
2. Slow execution if there are great contrasts in material stiffnesses or element sizes.

Strongly nonlinear systems



DYNAMIC RELAXATION

In dynamic relaxation gridpoints are moved according to Newton's law of motion. The acceleration of a gridpoint is proportional to the out-of-balance force. This solution scheme determines the set of displacements that will bring the system to equilibrium, or indicate the failure mode.

There are two important considerations with dynamic relaxation:

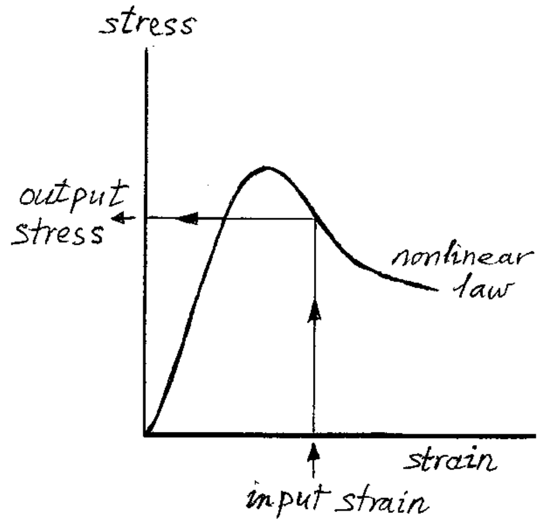
1) Choice of timestep
2) Effect of damping

A GENERAL FINITE-DIFFERENCE FORMULA

In the finite difference method, each derivative describing motion & stress-strain is replaced by an algebraic expression relating variables at specific locations in the grid.

The algebraic expressions are fully explicit; all quantities on the right-hand side of the expressions are known. Consequently each element (zone or gridpoint) in a *FLAC* grid appears to be physically isolated from its neighbors during one calculational timestep.

(The time-step is sufficiently small that information cannot propagate between adjacent elements during one step)



This is the basis of the calculation cycle:

DAMPING

Velocity-proportional damping introduces body forces that can affect the solution.

Local damping is used in *FLAC3D* --- The damping force at a gridpoint is proportional to the magnitude of the unbalanced force with the sign set to ensure that vibrational modes are damped:

LOCAL DAMPING

- **Damping forces are introduced to the equations of motion:**

$$\Delta \dot{u}_i = \left[\Sigma F_i - \alpha \, |\Sigma F_i| \, \text{sgn}\,(\dot{u}_i) \right] \frac{\Delta t}{m}$$

where F_i is the unbalanced force

- The damping force, $\boldsymbol{F_d}$ is:

$$F_d = -\alpha \left| \Sigma F_i \right| \text{sgn}(\dot{u}_i)$$

- **In *FLAC3D* the unbalanced force ratio (ratio of unbalanced force, F_i, to the applied force magnitude, F_m) is monitored to determine the static state.**

- **By default, when F_i / F_m < 0.00001 (1e-5), then the model is considered to be in an equilibrium state.**

STATIC ANALYSIS

FLAC3D is a dynamic solution method that provides a static solution (with the effect of inertial forces minimized) provided the unbalanced force ratio reaches a small value (~ 0.001 or less).

This is comparable to the "level of residual error" or "convergence criterion" defined for matrix solution methods used in many finite element programs. In *FLAC3D*, the level of error is quantified by the unbalanced force ratio. In both *FLAC3D* and FE solutions, the static solution process terminates when the error is below a desired value.

Calculation (stepping) summary

1. Nodal forces are calculated from stresses, applied loads and body forces (velocity and displacement vary linearly; stress and strain are constant within an element).
2. The equations of motion are invoked to derive new nodal velocities and displacements.
3. Element strain rates are derived from nodal velocities.
4. New stresses are derived from strain rates, using the material constitutive law.